The Digital Euro and Deposit-Market Competition

Portfolio Choice, Bank Switching, and Funding Adjustment

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Abstract

A non-remunerated central bank digital currency can change deposit competition even when households retain their bank accounts. I study a capped public-money option in a differentiated-bank model in which households choose both a bank and the division of liquid wealth between deposits and digital currency. Residual deposit demand responds through portfolio adjustment and bank switching; the latter depends on the second moment of deposit balances. This decomposition gives a conditional, rather than uniformly positive, deposit-rate response. With homogeneous customers and binding caps, a higher cap reduces the value of switching banks and lowers the equilibrium rate. I prove this result globally, show that it survives random-utility bank choice, and establish conditions under which wealth heterogeneity reverses it. A separate lending and wholesale-funding extension introduces an opposing scarcity effect and solves the joint deposit-pricing and lending response. Deposit markdowns, monetary-policy pass-through and household surplus need not move together. Piecewise-polynomial calculations verify all unilateral rate deviations in the numerical examples. Recent European and Indian evidence motivates the mechanisms without being treated as a calibration. The digital euro's institutional setting is assessed through 18 September 2026, with legislative proposals, pilot activity and actual issuance kept distinct.

Keywords: digital euro; central bank digital currency; deposit competition; bank switching; holding limits; wholesale funding.

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1 Introduction

A digital payment option is not necessarily an alternative banking relationship. A household can keep its current account, use public digital money for some transactions, and reduce the balance left at its bank. The bank then faces two responses to its deposit offer: customers can change their balances, or they can move to another bank. A theory of competition that collapses those responses into one aggregate deposit-demand curve can get the pricing effect wrong.

This paper asks how a capped, non-remunerated central bank digital currency (CBDC) changes that pricing problem. The digital euro provides the institutional motivation, but not an empirical treatment already observed in euro-area banking. The central analytical object is a bank's residual demand for deposits when household portfolio choice and bank choice are jointly determined. The public instrument need not pay interest to enter the portfolio: convenience services can make positive balances desirable. Conversely, positive uptake does not by itself make deposits more rate-sensitive.

The main result is a decomposition. At a symmetric rate, write μ for average deposits per household, Q for the response of those deposits to a common bank-rate change, and M_2 for their second moment. In the baseline spatial model, a bank's own-rate semi-elasticity is

$$S = \frac{Q}{\mu} + \frac{M_2}{\tau\mu},$$

where τ measures differentiation between banking relationships. The first term is portfolio substitution; the second is switching between banks. The second moment matters because a large-balance household both values a rate improvement more and transfers more deposits when it moves. At an interior equilibrium with constant marginal funding value a, the deposit markdown is $a - r = 1/S$.

This identity changes the interpretation of holding limits. When a cap binds, increasing it reduces bank balances without creating a local portfolio response to the deposit rate. With otherwise identical households, the switching component falls and banks offer a lower rate. I establish this as a global equilibrium result, not merely as a first-order condition. The mechanism also survives a logit specification of bank choice. It is not a consequence of having assumed an idiosyncratic form of aggregate demand.

The result is not universal. Two extensions identify precise reasons for the sign to change. First, with sufficiently dispersed wealth, an equal reduction in each household's deposits shifts the balance-weighted composition of demand towards large, rate-sensitive customers. An explicit sufficient condition makes the resulting symmetric solution a global equilibrium. Second, if lost deposits are replaced by increasingly expensive wholesale funding, the marginal value of retaining deposits rises. A tractable lending extension separates this scarcity effect from the change in demand elasticity and solves both deposit pricing and credit adjustment.

These mechanisms also distinguish three comparisons often treated as interchangeable. A smaller markdown is a statement about the level of a rate relative to funding value. Pass-through concerns the derivative of an equilibrium rate with respect to a common funding-value change. Household surplus includes convenience services as well as interest income. The paper gives examples and analytical results in which these objects do not move together. In particular, deposit rates can fall while households gain from a looser cap, and local pass-through can exceed one in a well-defined interior portfolio regime.

The approach builds on spatial differentiation (Hotelling 1929), discrete-choice demand (Train 2009), and the literature on CBDC competition. Andolfatto (2021) obtains deposit expansion through participation and saving responses to improved bank offers. Chiu et al. (2023) embeds deposit-market power in a general-equilibrium payments model and finds that CBDC competition can expand intermediation. Keister and Sanches (2023) instead emphasise a trade-off between exchange efficiency and bank funding. The present model does not nest all of these environments. Its narrower contribution is to make simultaneous wallet use and bank retention explicit, derive the corresponding demand moments, and state the conditions behind competing pricing predictions.

Recent evidence makes those distinctions relevant. Albertazzi et al. (2026) studies euro-area deposit pricing over 2007–2024 and associates weak sight-deposit pass-through with changes in the composition of depositors. In a different institutional setting, Di Maggio et al. (2026) reports deposit displacement following India's live retail-CBDC rollout. These findings motivate heterogeneity and funding adjustment; neither identifies the effect of a digital euro or supplies the parameters used below.

The project originates in my undergraduate dissertation (Ruzzante 2024), which reviewed the digital euro, banking and monetary policy using information available in June 2024. The present paper replaces that broad survey structure with a defined pricing game, proofs, a funding extension and a reproducible numerical analysis. It retains the dissertation's concern with banking and public digital money without attributing the later model or later evidence to the original work.

2 Institutional setting and the relevant evidence

2.1 The digital euro at the information cutoff

The proposed digital euro is a retail liability of the Eurosystem, distributed through supervised intermediaries. The proposed non-remuneration and individual holding limits distinguish it from an unrestricted public savings account. These are design features of a proposed instrument, not evidence that it has already been issued (European Commission 2023). Nor is a CBDC defined by the use of a particular blockchain: the identity of the issuer and the nature of the claim must be separated from its transfer technology (Bech and Garratt 2017).

On 30 October 2025 the ECB announced the transition beyond the 2023–25 preparation phase. Its objective of readiness for a possible first issuance during 2029 was conditional on legislation being adopted during 2026; an issuance decision would follow the legislation rather than be implied by technical preparation (ECB 2025a). The Council adopted a negotiating position on 19 December 2025. The European Parliament authorised negotiations on 9 July 2026, following the June decision of its Economic and Monetary Affairs Committee. Negotiating mandates are not the final regulation (Council of the European Union 2025; European Parliament 2026).

Pilot preparation has advanced separately. The ECB selected 36 payment service providers in July 2026 for a twelve-month pilot envisaged for the second half of 2027; it opened a further call for online and mobile merchants on 15 September 2026 (ECB 2026b; ECB 2026a). The pilot's limited participants and beta instrument must not be confused with general access to an issued CBDC.

The pilot's legal structure also needs care. The FAQ states that issuing the beta instrument against PSP funding creates Eurosystem liabilities. Its online form is scriptural money under the payments framework, not a direct account with a Eurosystem central bank; participants also use commercial-bank-money accounts with pilot providers (ECB 2026c). Thus, the absence of a direct central-bank account does not imply an absence of Eurosystem liabilities. Deposit displacement must be assessed by tracing participant balances, PSP funding and the public liability, not inferred from the account label.

2.2 Which design features enter the model?

The paper maps proposed features to economic primitives without assuming that technical design determines those primitives mechanically. The baseline sets CBDC remuneration r^m to zero and imposes a household balance cap $\bar{m}$. Convenience preferences capture the private value of using each instrument. Bank differentiation captures the cost or inconvenience of changing the banking relationship. The latter is not the same thing as wallet onboarding or payment acceptance.

The envisaged funding links matter for that separation. A waterfall moves receipts above the permitted wallet balance to a linked bank account; a reverse waterfall draws on that account when a payment exceeds available digital euro holdings. These mechanisms separate the stock of public money from the value of payments it can support. The ECB's design also envisages immediate merchant defunding rather than unrestricted merchant accumulation (ECB 2025b). In the static model, convenience is a reduced-form benefit of holding balances. Instant funding can lower the balances required for a given payment service, so the model should not be read as imposing proportionality between wallet balances and payment volume.

The numerical cap remains a parameter, not an adopted policy value. At the request of co-legislators, the ECB assessed hypothetical limits from €500 to €3,000. Its closing report expressly distinguishes that exercise from a final view on the appropriate holding limit (ECB 2025b). The simulations below therefore use normalised balance units. They neither estimate a euro-denominated limit nor convert a scenario in an official assessment into a policy decision.

2.3 Evidence does not identify a single competition effect

Three kinds of evidence address different parts of the mechanism. Euro-area deposit-pricing estimates document heterogeneity in rate sensitivity and the movement of more responsive customers towards higher-yielding savings products. In Albertazzi et al. (2026), the remaining sight-deposit pool becomes less rate-sensitive. That is evidence about private deposit markets before retail-CBDC issuance, not a measured CBDC effect.

Household survey experiments address intended use and hypothetical allocation. Georgarakos et al. (2025) reports stated willingness to use the digital euro and generally small allocation responses over a range of hypothetical caps. The estimates are not uniformly zero: the €1,000 limit reduces allocated liquid wealth in some specifications. Hypothetical choices, realised balances and responses during a run are distinct outcomes. The study consequently motivates heterogeneous cap exposure, not an assumption that caps never matter.

In their August 2026 revision, Di Maggio et al. (2026) reports a 2.7% reduction in retail deposits associated with CBDC eligibility at a large Indian public-sector bank, concentrated in liquid savings accounts. The authors also report a temporary credit decline and substitution away from deposit-backed digital payments rather than cash. These are findings in the Indian

rollout, not euro-area treatment effects. They motivate separate measurement of deposit quantities, funding replacement and payment choices.

Balance-sheet exercises provide a fourth, conditional perspective. Auer et al. (2024) examines Italian banks under hypothetical deposit substitution; Becker and Grabia (2025) studies euro-area holding limits using bank-level information and simulated account-balance distributions. These analyses emphasise heterogeneous funding capacity. They do not estimate the derivative of an individual bank's deposit demand with respect to its own rate. That derivative remains the pricing object to be derived.

3 Household portfolios and differentiated banks

3.1 Environment and timing

A unit mass of households has type $z = (w, \theta, \phi)$, with a fixed distribution F of compact support. Liquid wealth satisfies $0 < \underline{w} \leq w \leq \overline{w}$; the convenience weights θ and ϕ are strictly positive and bounded. The CBDC cap obeys $0 \leq \bar{m} < \underline{w}$. This restriction ensures positive deposits for every household, so the baseline studies simultaneous bank use and CBDC use rather than exit from banking.

Independently of z, a household has location $x \sim U[0, 1]$. Banks A and B are located at the endpoints. Using their services entails costs τx and $\tau(1 - x)$, respectively, with $\tau > 0$. Locations represent differentiation in services or relationships; they need not be geographical. There is no previous-period bank choice. Thus τ is a static representation of the cost of moving between offers, not an estimated dynamic switching cost.

CBDC design and household characteristics are fixed first. Banks then choose net deposit rates simultaneously. Households observe both offers, choose one bank, and allocate their liquid wealth between deposits and CBDC. All rates and convenience values refer to the same model period. The numeraire is consumption at that period's end; the common principal w can be omitted from utility.

The baseline assumes a constant net marginal private value $a > 0$ of deposit funding and a bank rate set $[0, a]$. It excludes fees, risk differences between banks, endogenous saving and lending feedback. Section 7 replaces the constant funding value with a loan and wholesale-funding problem. The non-negative rate restriction is an explicit boundary of the baseline, not an assertion that negative deposit rates are impossible in other settings.

3.2 The portfolio problem

For a bank offering rate r, define the household's indirect portfolio value as

$$V_z(r; r^m, \bar{m}) = \max_{0 \leq m \leq \bar{m}} \{r(w - m) + r^m m + \theta v(w - m) + \phi v(m)\}. \tag{1}$$

Here m is CBDC and $d = w - m$ is deposits. The function v is increasing, twice continuously differentiable and strictly concave on $[0, \overline{w}]$. Assume that

$$-\theta v''(w - m) - \phi v''(m) \geq \underline{K} > 0$$

throughout the feasible type and balance set. These assumptions give a unique portfolio and bounded local responses. The baseline $r^m = 0$ reflects the proposed digital euro. Changes in r^m below are counterfactual comparative statics, not forecasts of remuneration policy.

Write

$$g_z(m; r) = r^m - r - \theta v'(w - m) + \phi v'(m). \tag{2}$$

For $\bar{m} > 0$, strict concavity yields the complete optimality conditions

$$\begin{aligned} m_z = 0 &\iff g_z(0; r) \leq 0, \\ 0 < m_z < \bar{m} &\iff g_z(0; r) > 0 > g_z(\bar{m}; r), \\ m_z = \bar{m} &\iff g_z(\bar{m}; r) \geq 0. \end{aligned} \tag{3}$$

In the middle case, $g_z(m_z; r) = 0$. At $\bar{m} = 0$, feasibility alone determines $m_z = 0$. Equalities at the corner thresholds may create kinks in balances; the inequalities are not interchangeable with first-order conditions for an interior optimum.

When the portfolio is interior, define

$$K_z = -\theta v''(d_z) - \phi v''(m_z) > 0. \tag{4}$$

Implicit differentiation gives

$$d_{z,r} = K_z^{-1}, \qquad d_{z,r^m} = -K_z^{-1}, \qquad d_{z,\bar{m}} = 0. \tag{5}$$

At a strict upper-cap corner, $d_z = w - \bar{m}$, so the first two derivatives are zero and $d_{z,\bar{m}} = -1$. At a strict zero-CBDC corner, all three derivatives are zero. A higher cap therefore changes balances only for constrained types at a fixed rate; it does not automatically create a positive local response to remuneration.

The interior equality can also be written

$$r - r^m = \phi v'(m_z) - \theta v'(d_z).$$

It supplies no unconditional ordering of the two interest rates. Convenience services can compensate for a lower return. In particular, positive holdings of a non-remunerated CBDC do not require households to be indifferent to interest income.

Uniqueness of the portfolio optimum gives the envelope identity $V_{z,r} = d_z$, including at balance kinks. For derivatives of aggregate demand, assume no positive mass of types is exactly at a portfolio threshold at the rate in question. Bounded wealth and curvature then permit differentiation under the integral. When second-order aggregate derivatives are required, the relevant moments will additionally be assumed continuously differentiable on a neighbourhood. No differentiability across a mass of switching types is presumed.

3.3 Bank choice and residual demand

A type-z household selects bank A if

$$V_z(r_A) - \tau x \geq V_z(r_B) - \tau(1 - x).$$

Its share at that bank is therefore

$$s_A(z; r_A, r_B) = \mathrm{clip}_{[0,1]}\left[\frac{1}{2} + \frac{V_z(r_A) - V_z(r_B)}{2\tau}\right], \tag{6}$$

where $\mathrm{clip}_{[0,1]}(q) = \min\{1, \max\{0, q\}\}$. Ties involve a set of locations of measure zero. Deposits and profits are

$$D_A(r_A, r_B) = \int d_z(r_A) s_A(z; r_A, r_B)\, \mathrm{d}F(z), \qquad \pi_A = (a - r_A) D_A. \tag{7}$$

Bank B is defined symmetrically. An equilibrium is a pair of rates each maximising its bank's profit over the entire interval $[0, a]$, given the other rate and household optimisation. This global definition remains in force when demand is differentiable locally.

The cost of a banking relationship is paid per household, rather than per euro. A household with a larger deposit balance consequently gains more from a given rate difference. That assumption is economically consequential and will be varied explicitly. Fixed participation and a fixed liquid portfolio also mean that the model cannot generate new saving or entry into banking. With no CBDC, all of the earmarked liquid wealth is already deposited.

4 Residual demand and the deposit markdown

At a symmetric rate r, define three moments:

$$\mu = \mathbb{E}[d_z(r)], \qquad Q = \mathbb{E}[d_{z,r}(r)], \qquad M_2 = \mathbb{E}[d_z(r)^2]. \tag{8}$$

Every type splits equally between banks. Bounded wealth implies that a sufficiently small unilateral deviation leaves all type-specific shares strictly between zero and one.

Proposition 1 (Portfolio adjustment and bank switching)**.** *At a differentiable symmetric rate, bank A's deposits and rate derivatives are*

$$D_A = \frac{\mu}{2}, \qquad D_{A,r_A} = \frac{Q}{2} + \frac{M_2}{2\tau}, \qquad D_{A,r_B} = -\frac{M_2}{2\tau}. \tag{9}$$

Any symmetric interior pure-strategy Nash equilibrium satisfies

$$a - r^* = \frac{\mu}{Q + M_2/\tau} = \frac{1}{S}, \qquad S := \frac{Q}{\mu} + \frac{M_2}{\tau\mu} > 0. \tag{10}$$

All moments in the pricing equation are evaluated at r^*.

Proof. The envelope theorem and (6) give $s_{A,r_A} = d_z/(2\tau)$ and $s_{A,r_B} = -d_z/(2\tau)$ at symmetry. Differentiating the product $d_z s_A$ gives (9). Differentiation of profit yields $\pi_{A,r_A} = -D_A + (a - r_A) D_{A,r_A}$. Its vanishing at an interior optimum gives (10). □

The first term of the own-rate derivative captures larger balances held by current customers; the second captures customers arriving from the rival. This is why an aggregate deposit-demand curve is not a bank's residual demand. If both banks raise their rates by the same amount, household bank shares remain unchanged and total deposits respond by Q. A bank acting alone also reallocates existing deposits between institutions.

The implied demand Jacobian is

$$J = \frac{1}{2}\begin{pmatrix} Q + M_2/\tau & -M_2/\tau \\ -M_2/\tau & Q + M_2/\tau \end{pmatrix}. \tag{11}$$

Its common-rate and relative-rate eigenvalues are $Q/2$ and $Q/2 + M_2/\tau$. These distinct directions explain why a large bank-specific response need not imply large aggregate substitution into or out of the public instrument.

The term $M_2/\mu = \mu + \mathrm{Var}(d_z)/\mu$ is a balance-weighted average balance. Holding μ, Q and τ fixed, greater dispersion raises the switching contribution. This is not an unconditional comparative static with respect to inequality: changing the distribution of wealth can also change portfolio regimes, Q, and the equilibrium rate. It identifies the moment that would need to be held fixed or measured.

Equation (10) is an inverse semi-elasticity condition. Calling it a conventional Lerner index would obscure that the bank is buying funding by paying a rate, rather than selling a good at a price. It also does not identify $a - r$ with an observed bank-wide net interest margin: a is a marginal funding value and the baseline omits fees and other activities.

Differentiation between banks is essential to the positive markdown. Under otherwise homogeneous, perfectly substitutable bank offers, the highest rate attracts all deposits and the standard undercutting logic eliminates a positive funding margin, provided deposits remain positive at a. The smooth share rule in (6) supplies the differentiation that makes the derivative meaningful. It is not consistent to assume both frictionless winner-take-all bank choice and a smooth residual demand without an additional source of heterogeneity.

4.1 Local comparative statics are not existence proofs

Let λ be a design parameter other than a, and write

$$H(r, \lambda) = r + S(r, \lambda)^{-1} - a.$$

On a continuously differentiable symmetric branch with $S^2 - S_r > 0$, the implicit-function theorem gives

$$r^*_\lambda = \frac{S_\lambda}{S^2 - S_r}, \qquad r^*_a = \frac{S^2}{S^2 - S_r}. \tag{12}$$

Here S_r differentiates the semi-elasticity evaluated at a *common* rate, while the semi-elasticity itself concerns an *own*-rate deviation. The denominator condition describes a regular branch. It is not a theorem that bank profits are concave or that all other deviations are unprofitable.

An improvement in CBDC convenience can reduce deposits at a fixed rate without having a determinate effect on S. The ratio Q/μ may rise, while $M_2/(\tau\mu)$ may fall. Holding limits further alter which types are in the responsive interior regime. Strict concavity of household utility is sufficient for portfolio uniqueness, but is not sufficient to sign these competing demand effects or to establish global concavity of bank profit.

The subsequent sections therefore separate three levels of result: identities valid at a symmetric rate; local derivatives conditional on an equilibrium branch; and global equilibrium results under explicit restrictions. The numerical procedure checks the last requirement directly in every reported example.

5 Binding caps, wealth dispersion and household surplus

5.1 A globally solved cap-binding economy

Consider first identical portfolio types (w, θ, ϕ). Suppose $\bar{m} > 0$ and

$$r^m - a - \theta v'(w - \bar{m}) + \phi v'(\bar{m}) \geq 0. \tag{13}$$

Since g decreases in r, this condition makes the cap bind for *every* feasible deposit offer, not just at a conjectured equilibrium. Put $d = w - \bar{m} > 0$. If $\bar{m} = 0$, deposits equal w by feasibility and the same pricing analysis applies without (13).

Proposition 2 (Global equilibrium with identical capped portfolios)**.** *Under these conditions there is a unique symmetric equilibrium,*

$$r^* = \max\left\{0, a - \frac{\tau}{d}\right\}, \qquad D_A = D_B = \frac{d}{2}. \tag{14}$$

If $ad > \tau$, each bank earns $\tau/2$. On this interior branch, at fixed τ,

$$\frac{\partial r^*}{\partial \bar{m}} = -\frac{\tau}{d^2} < 0. \tag{15}$$

If $ad \leq \tau$, the symmetric rate is zero and each bank earns $ad/2$.

The proof in Appendix B checks the complete deviation interval, including rates at which a bank loses all customers or captures the entire market. The zero-rate branch prevents the interior expression from being extrapolated to negative rates outside the strategy set.

The mechanism is not a fall in households' appreciation of payment services. When all wallets are capped, a small increase in the deposit rate cannot recover CBDC balances. It can only attract another bank's customers. A household holding d deposits gains $d\,\Delta r$ from a rate improvement; reducing d weakens that incentive relative to a fixed relationship cost. The markdown rises by exactly enough to offset the smaller deposit base, leaving interior bank profits unchanged.

This provides a counterexample to an unconditional statement that a public digital-money option raises deposit rates or lowers bank profits. It does not imply that a digital euro would necessarily do the reverse. The result depends on full bank participation, a binding cap, fixed relationship differentiation and a constant marginal funding value. Each of those restrictions is visible in the solution.

5.2 Interoperability and the scale of relationship costs

Suppose an accompanying change in services reduces differentiation, represented by a differentiable $\tau(\bar{m}) > 0$. On the same interior branch,

$$\frac{\mathrm{d}r^*}{\mathrm{d}\bar{m}} = -\frac{\tau'(\bar{m})}{d} - \frac{\tau(\bar{m})}{d^2}. \tag{16}$$

Rates rise exactly when $-\tau'/\tau > 1/d$. The proportional reduction in differentiation must exceed the proportional reduction in deposits per household. This is a threshold calculation, not an assertion that wallet interoperability produces account portability. Wallet-provider choice and deposit-bank choice must be measured separately.

The scale of relationship costs also matters. Within the fully constrained economy, replacing the fixed cost scale by $\tau_0 d^\eta$ gives

$$r^* = a - \tau_0 d^{\eta-1}, \qquad r^*_{\bar{m}} = (\eta - 1)\tau_0 d^{\eta-2}$$

on the positive-rate branch. A proportional cost ($\eta = 1$) eliminates the cap effect on the rate; subproportional and superproportional costs give opposite signs. For $\eta \geq 0$, condition (13) remains sufficient for cap binding even if the household accounts for balance-dependent relationship costs: reducing deposits weakly reduces that cost. This comparison belongs to the constrained economy; it is not substituted into the unconstrained envelope calculation in Section 4.

5.3 Wealth dispersion can reverse the result

Now let wealth vary while every type remains at the cap for every feasible rate. Define $\mu = \mathbb{E}[w] - \bar{m}$, $v_w = \mathrm{Var}(w)$ and $d_{\max} = \overline{w} - \bar{m}$. The candidate interior rate is

$$r^* = a - \frac{\tau\mu}{\mu^2 + v_w}. \tag{17}$$

Dispersion is unchanged when the same cap is subtracted from every household's wealth, but its importance relative to mean deposits increases.

Proposition 3 (A global heterogeneous cap equilibrium)**.** *Assume cap binding throughout* $[0, a]$ *for all types. If the rate in* (17) *satisfies*

$$0 < r^* \quad \textit{and} \quad r^* d_{\max} \leq \tau, \tag{18}$$

then (r^*, r^*) *is a global Nash equilibrium and the unique symmetric equilibrium. For cap changes preserving these conditions,*

$$r^*_{\bar{m}} = \frac{\tau(v_w - \mu^2)}{(\mu^2 + v_w)^2}. \tag{19}$$

The second condition rules out losing an entire type on a downward rate deviation. It is sufficient, not necessary. Upward deviations can saturate type-specific shares; the proof deals with them rather than assuming them away. Thus the formula is not an unsupported first-order solution.

The derivative is positive when the coefficient of variation of deposits exceeds one. An equal absolute reduction in balances removes a larger proportion from small accounts, increasing the relative importance of high-balance customers in residual demand. This result gives the second moment in Proposition 1 an equilibrium interpretation. It also shows why the homogeneous result should not be extrapolated from the mean account balance alone. Section 9 gives a finite-type example satisfying the global condition.

5.4 A lower deposit rate need not reduce household surplus

In the homogeneous interior cap equilibrium, average household utility, apart from common principal, is

$$\mathcal{U}(\bar{m}) = r^* d + r^m \bar{m} + \theta v(d) + \phi v(\bar{m}) - \frac{\tau}{4}. \tag{20}$$

The last term is the expected cost of using the nearer bank. At a fixed τ,

$$\mathcal{U}'(\bar{m}) = r^m - a - \theta v'(d) + \phi v'(\bar{m}) \geq 0, \tag{21}$$

where the inequality is exactly the global cap-binding condition. The gain from relaxing a constraint offsets both the loss of interest on shifted balances and the lower equilibrium rate. Interior aggregate bank profit is constant, so this result also describes the change in combined household and bank surplus in this special economy.

This is deliberately not a social-welfare theorem about issuing a digital euro. Public operating costs, the return on public backing assets, merchant services, distributional weights and financial-stability externalities are absent. Its purpose is narrower: even the sign of household benefit cannot be inferred from the sign of the deposit-rate response. Outside the strong cap-binding region, expression (21) is not a general formula.

6 What changes under random-utility choice?

A discrete-choice benchmark helps separate the role of a public outside option from the role of a capped portfolio asset. The two are not interchangeable. Logit demand is a standard consequence of independent extreme-value taste shocks (Train 2009, Chapter 3); the economic issue is which alternatives those shocks compare.

6.1 Choosing a bank while retaining a CBDC wallet

Replace spatial differentiation with $N \geq 2$ banks and utility $V_z(r_j) + \varepsilon_j$, where independent taste shocks have scale $\sigma > 0$. The household still divides its wealth as in (1) after selecting a bank. Every household uses one bank; the wallet is available with each bank. At symmetry,

$$S_N = \frac{Q}{\mu} + \left(1 - \frac{1}{N}\right) \frac{M_2}{\sigma\mu}. \tag{22}$$

Appendix C derives this identity. The same two moments determine competition, with a different coefficient on switching.

For homogeneous, globally capped portfolios, $Q = 0$ and $M_2 = d^2$. If $ad > N\sigma/(N-1)$, there is a global symmetric interior equilibrium

$$r^* = a - \frac{N\sigma}{(N-1)d}, \qquad \pi_j = \frac{\sigma}{N-1}, \qquad r^*_{\bar{m}} = -\frac{N\sigma}{(N-1)d^2} < 0. \tag{23}$$

The proof establishes global optimality of each bank's offer. Hence the negative cap derivative is not peculiar to linear city shares. It follows from retaining the banking relationship while reducing the balance over which an interest-rate gain is earned.

The number of banks affects the coefficient but does not remove differentiation when N grows: under this logit scaling, idiosyncratic tastes remain. Neither $\tau \to 0$ nor $\sigma \to 0$ should be interpreted as a regular differentiated equilibrium at the limiting point without checking the resulting discontinuous-choice problem.

6.2 A mutually exclusive public-money alternative

For comparison, suppose a household places a fixed bundle $\ell > 0$ either at bank A, at bank B, or entirely in a public alternative. Bank utility is $q + \ell r_j + \varepsilon_j$ and public-option utility is $A_0 + \varepsilon_0$, with shock scale σ. Let $\alpha = \ell/\sigma$. At symmetry, each bank has share s and the public option has share $s_0 = 1 - 2s$. Bank deposits are ℓs, and the interior pricing condition is

$$a - r^* = \frac{1}{\alpha(1-s)}. \tag{24}$$

Assume $a > 2/\alpha$, which guarantees an interior symmetric rate for every finite A_0. There is a unique symmetric equilibrium, and a global own-rate best-response check validates the first-order solution.

In this benchmark a more attractive public alternative raises deposit rates:

$$\frac{\partial r^*}{\partial A_0} = \frac{ss_0}{\ell\{(1-s)^2 + ss_0\}} > 0, \qquad \frac{\partial r^*}{\partial a} = \frac{(1-s)^2}{(1-s)^2 + ss_0} \in (0,1). \tag{25}$$

The bank share and bank profit $\sigma s/(1-s)$ fall, while expected household utility rises. Appendix D provides the proof and the surplus calculation.

The comparison does not contradict the portfolio model. Here switching to the public option removes the entire bundle from banking. In the main model a household retains a bank account and changes its balance. Moreover, a cap below ℓ makes the exclusive public allocation infeasible. Replacing A_0 by an increasing function of the cap does not enforce that quantity constraint. The exclusive-choice benchmark is useful for understanding outside-option competition, but it cannot establish a cap comparative static for joint wallet and bank use merely by relabelling its quality parameter.

These distinctions also discipline interpretation of the literature. A public option can improve bank offers in an environment with an extensive margin, while a capped balance substitute can reduce residual rate sensitivity under full bank participation. The source of the difference is the household choice set and the resulting demand derivative, not an inconsistency between the pricing first-order conditions.

7 Endogenous funding value and bank lending

The baseline assigns a constant marginal value a to deposits. That is useful for isolating demand, but it rules out a rise in funding scarcity after deposit losses. This section adds a bank-specific lending opportunity and wholesale replacement. It remains partial equilibrium: there is no common loan-market-clearing condition, interbank contagion or endogenous monetary-policy response.

7.1 The bank's funding problem

After raising deposits D, a bank chooses lending $L \geq D$ and wholesale funding $F_b = L - D \geq 0$. Its net value before deposit interest is

$$\Psi(D) = \max_{L \geq D} \{R(L) - C(L - D)\}, \tag{26}$$

where R is net loan revenue after non-funding costs and C is wholesale funding cost. Both are measured net of principal. Assume $R'' < 0$, $C'' \geq 0$, and a unique interior optimum with $F_b > 0$ on the relevant deposit interval. The envelope theorem yields

$$\lambda(D) := \Psi'(D) = C'(F_b) = R'(L), \qquad \lambda'(D) = \frac{R''C''}{C'' - R''} \leq 0. \tag{27}$$

The bank chooses its deposit rate to maximise $\Psi(D_j) - r_j D_j$. Consequently, at a symmetric interior rate,

$$\lambda(\mu/2) - r^* = \frac{1}{S}. \tag{28}$$

The same demand moments determine the markdown, but its reference value now changes with deposits. This is the distinction between stronger competitive pressure and a higher marginal value of funding.

Differentiating with respect to the cap, with $\mu_{\bar{m}}$ and $S_{\bar{m}}$ evaluated at a fixed rate,

$$r^*_{\bar{m}} = \frac{S_{\bar{m}}/S^2 + \lambda'(\mu/2)\mu_{\bar{m}}/2}{1 - S_r/S^2 - \lambda'(\mu/2)Q/2}, \tag{29}$$

provided the denominator is positive and the solution remains a global equilibrium. Since both λ' and $\mu_{\bar{m}}$ are non-positive, the direct funding-scarcity term in the numerator is non-negative. It can oppose a fall in residual semi-elasticity.

7.2 A closed-form lending and rate equilibrium

Take

$$R(L) = AL - \frac{kL^2}{2}, \qquad C(F_b) = c_0 F_b + \frac{\chi F_b^2}{2}, \qquad k, \chi, c_0 > 0. \tag{30}$$

For homogeneous capped deposits $d = w - \bar{m}$, require $A - c_0 > kd$. This keeps wholesale funding strictly positive for every bank deposit volume $D \in [0, d]$. Direct solution gives

$$\begin{aligned} L(D) &= \frac{A - c_0 + \chi D}{k + \chi}, \quad F_b(D) = \frac{A - c_0 - kD}{k + \chi}, \\ \lambda(D) &= \lambda_0 - bD, \qquad\qquad \lambda_0 = \frac{kc_0 + \chi A}{k + \chi}, \qquad b = \frac{k\chi}{k + \chi} > 0. \end{aligned} \tag{31}$$

Let banks choose rates in $[0, \lambda_0]$. Replace a by λ_0 in the sufficient portfolio condition (13), so the cap binds under every permitted offer.

Proposition 4 (Funding scarcity and lending adjustment)**.** *If $d\lambda(d/2) > \tau$, the rate game has a global symmetric interior equilibrium*

$$r^* = \lambda_0 - \frac{bd}{2} - \frac{\tau}{d}. \tag{32}$$

It is the unique symmetric equilibrium. On a cap-binding neighbourhood preserving the assumptions,

$$r^*_{\bar{m}} = \frac{b}{2} - \frac{\tau}{d^2}, \qquad L^*_{\bar{m}} = -\frac{\chi}{2(k + \chi)}, \qquad F^*_{b,\bar{m}} = \frac{k}{2(k + \chi)}. \tag{33}$$

The proof in Appendix E compares all rate deviations. Since Ψ is concave, profit is strictly concave on the interval with interior customer shares; zero- and full-share regions are checked

separately. Thus introducing the lending margin does not rely on an unsupported assumption that the first-order solution is globally optimal.

The rate derivative contains opposing terms. The fall in balances weakens bank switching by τ/d^2. The rising marginal value of deposits adds $b/2$. Rates rise if funding scarcity dominates, even though every wallet remains capped and no household changes its portfolio in response to a small rate increase. A higher rate is therefore not, by itself, evidence that the public instrument made deposit demand more elastic.

A one-unit reduction in a bank's deposits reduces its lending by the fraction $\chi/(k+\chi)$ and raises replacement funding by $k/(k+\chi)$. These fractions sum to one. With perfectly elastic wholesale funding ($\chi = 0$), lending is unchanged on the interior branch and funding replacement is one-for-one. As wholesale costs become very steep, lending approaches a one-for-one response. These are comparative statics of the specified funding technology, not empirical estimates of credit contraction.

The extension also makes a restriction transparent. In the binding-cap economy, rate changes cannot restore aggregate deposits, because balances are fixed at d. Deposit competition reallocates those balances between banks, whereas the funding technology determines how banks absorb the common loss. Endogenous saving, participation and asset demand would be needed to generate additional aggregate funding. The participation and saving mechanisms in Andolfatto (2021) therefore lie outside this extension rather than being disproved by it.

8 Pass-through is a separate comparative static

Deposit-market power is relevant to monetary transmission, but the level of a markdown and the response of that markdown to a common funding-value change are different objects. The deposits-channel literature links deposit pricing and funding flows (Drechsler et al. 2017); subsequent work shows that empirical conclusions also depend on sample, market-power measurement and lending categories (Schaffer and Segev 2022). The present model identifies a static derivative, not the speed of adjustment to a monetary-policy shock.

In the constant-funding-value model, equation (12) implies $r_a^* = S^2/(S^2 - S_r)$. Thus a larger S alone does not establish stronger pass-through: its slope S_r also matters. Nor is a itself the ECB policy rate. If a policy variable i affects only funding value, then $\mathrm{d}r^*/\mathrm{d}i = r_a^* a_i$. Direct effects on convenience, CBDC remuneration or wholesale terms require additional terms.

A quadratic portfolio specification makes the distinction exact. Let

$$v(q) = q - \frac{\kappa q^2}{2}, \quad B = \kappa(\theta + \phi), \quad C = r^m + \phi - \theta + \kappa\theta w, \qquad 0 < \kappa w < 1. \tag{34}$$

For identical types,

$$m(r) = \min\left\{\bar{m}, \max\left\{0, \frac{C - r}{B}\right\}\right\}. \tag{35}$$

On an interior portfolio branch, put $b_p = 1/B$, $d = w - C/B + b_p r$ and $u = d^2/(b_p\tau) > 0$.

Proposition 5 (Interior portfolio pass-through)**.** *Within a homogeneous quadratic interior branch there is at most one symmetric stationary rate. Whenever it is an interior global equilibrium that persists on that branch,*

$$\beta := \frac{\partial r^*}{\partial a} = \frac{(1+u)^2}{u^2 + u + 2}, \qquad \frac{\partial r^*}{\partial r^m} = 1 - \beta. \tag{36}$$

The derivative satisfies $1/2 < \beta \leq 8/7$*. It is below one for* $u < 1$*, equals one at* $u = 1$*, and exceeds one for* $u > 1$*, reaching its maximum at* $u = 3$*.*

Appendix F proves both branch uniqueness and the bounds. The statement does not say that a stationary solution exists in every branch, or that it remains optimal after a constraint is crossed. In the cap-binding homogeneous branch of Proposition 2, pass-through with respect to a is instead exactly one while the rate remains positive.

The identity $r_a^* + r_{r^m}^* = 1$ is consistent with shifting the funding value and both relevant returns by the same amount on the interior branch. It also demonstrates that even counterfactual CBDC remuneration need not increase the deposit rate in this model. The rate response is determined by demand curvature and bank switching, not by a blanket claim that a more attractive alternative always disciplines pricing.

The scope remains limited. A non-remunerated digital euro does not add an actively adjusted CBDC interest-rate instrument to the model. Studies of monetary implementation and remunerated CBDC, such as Meaning et al. (2021), consider a broader environment. Cash convertibility, reserve supply, collateral and policy responses would have to be specified before translating the derivatives here into claims about an effective lower bound or the overall potency of monetary policy.

9 Numerical examples and global verification

The examples illustrate mechanisms, not their quantitative importance in the euro area. All parameters are synthetic. Balance units are normalised, and rates are net returns over the model period rather than estimates of annual market rates. The code constructs equilibria from household optimisation and the complete bank rate game; it does not calibrate an optimal digital euro cap.

9.1 Portfolio substitution can give either rate response

Table 1 uses (34) and compares no CBDC with a positive cap, holding all other primitives fixed within each column. The shared parameters are $w = 1$, $a = 0.04$, $\kappa = 0.5$ and $r^m = 0$.

In the first case, the global cap-binding result applies. In the second, the portfolio-response term is large enough that the public option raises the equilibrium deposit rate. In the third, deposits are also interior, but the loss of bank-switching sensitivity dominates. A common conclusion about the direction of deposit pricing would therefore be inconsistent even with these simple cases. The first-order condition alone does not certify any of them; the global checks are separate.

Figure 1 isolates the cap and differentiation channels. In both scenarios every wallet is capped. The dashed scenario additionally lowers τ as the cap changes. It is a joint change in primitives, not an estimated technological consequence of the digital euro. Figure 2 illustrates the exact local derivative in Proposition 5.

9.2 Composition and funding scarcity

For the heterogeneous example, 95% of households have $w = 1$ and 5% have $w = 10$. Set $\bar{m} = 0.5$, $a = 0.04$, $\tau = 0.15$, $\kappa = 0.05$, $\theta = 0.02$, $\phi = 0.12$ and $r^m = 0$. Every type is capped under every feasible offer. Mean deposits are 0.95, their variance is 3.8475, and their

Table 1: Synthetic equilibria with homogeneous portfolio types

	Case 1	Case 2	Case 3
θ	0.02000	0.03667	0.50000
ϕ	0.12000	0.06333	0.50000
τ	0.005	0.020	0.005
$\bar{m}$	0.30	0.95	0.60
Portfolio	Capped	Interior	Interior
Rate, no CBDC	0.03500	0.02000	0.03500
Rate, with CBDC	0.03286	0.02424	0.03139
Deposit balance	0.70000	0.58474	0.56277
CBDC balance	0.30000	0.41526	0.43723
Rate change (basis points)	-21.43	+42.37	-36.13
Bank profit	0.002500	0.004609	0.002423

Author's calculations. Every reported rate is checked against all unilateral rate deviations in $[0, a]$. "Interior" describes the household portfolio; the bank rate is also interior in all three examples.

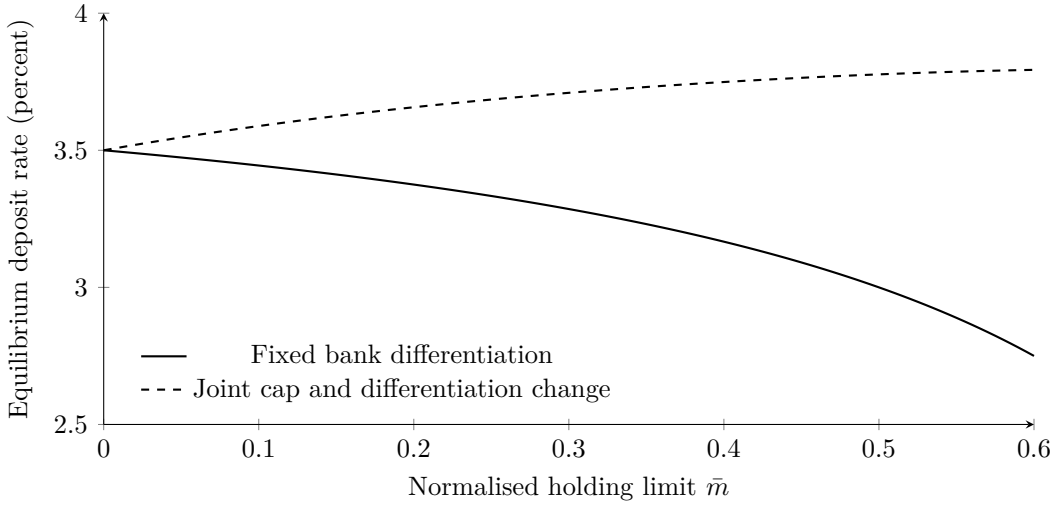


Figure 1: A cap increase and a simultaneous reduction in bank differentiation

Synthetic parameters: $w = 1$, $a = 0.04$, $\theta = 0.02$, $\phi = 0.12$, $\kappa = 0.5$, $r^m = 0$. The solid case holds $\tau = 0.005$ fixed; the dashed case sets $\tau = 0.005e^{-3\bar{m}}$. The global cap-binding condition and positive-rate condition hold over the displayed interval.

second moment is 4.75. The equilibrium rate is 0.01, and $r^* d_{\max} = 0.095 < 0.15$, verifying the sufficient global condition. The cap derivative is positive. This is a constructed distributional example, not a description of European account holdings.

The funding examples use $w = 1$, $\bar{m} = 0.3$, $\tau = 0.005$, $k = 0.04$, $c_0 = 0.01$ and $\lambda_0 = 0.08$, with A chosen from (31) to hold λ_0 fixed. Set $\theta = 0.02$, $\phi = 0.30$, $\kappa = 0.5$, and $r^m = 0$ to ensure global cap binding. Varying χ changes the scarcity slope b without changing the household demand primitives. Table 2 reports the contrasting derivatives.

9.3 The verification algorithm

With quadratic convenience, each type's deposit balance is constant or affine in its bank's rate, and its indirect utility is piecewise quadratic. For a fixed rival rate, customer shares

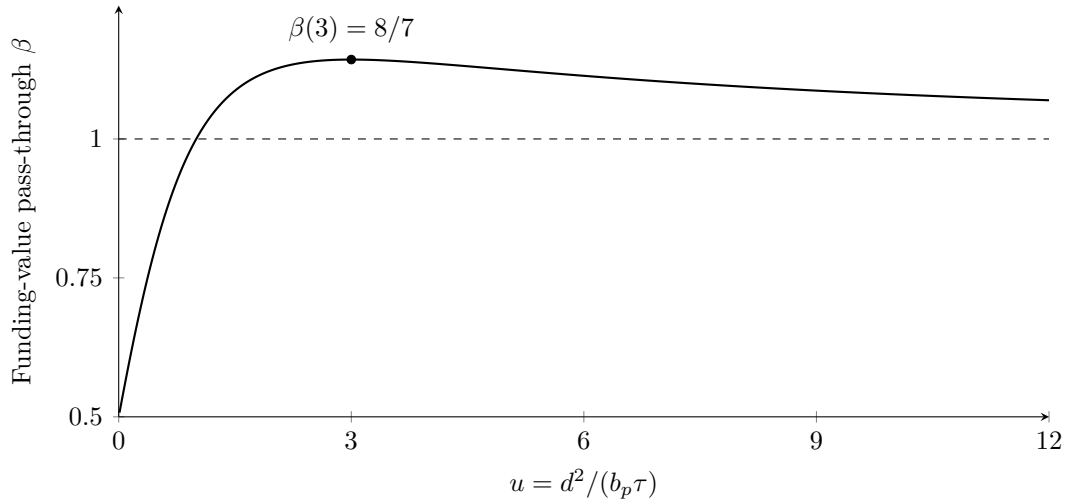


Figure 2: Funding-value pass-through on a quadratic interior portfolio branch

The curve is the analytical expression in Proposition 5, not a fitted response. Its argument $u = d^2/(b_p\tau)$ summarises balance size relative to portfolio responsiveness and differentiation. Values are conditional on existence and global optimality on the interior branch.

Table 2: Synthetic extensions: cap derivatives and lending adjustment

Example	r^*	$r^*_{\bar{m}}$	$L^*_{\bar{m}}$	$F^*_{b,\bar{m}}$
Heterogeneous, capped	0.01000	+0.01958	–	–
Funding, $\chi = 0.01$	0.07006	-0.00620	-0.10000	+0.40000
Funding, $\chi = 0.10$	0.06286	+0.00408	-0.35714	+0.14286

A derivative is per unit increase in the normalised cap. Lending and wholesale funding are per bank. Dashes indicate that the constant-funding-value model has no lending choice. These derivatives are not basis-point effects of a one-euro policy change.

become zero or one at roots of known quadratic polynomials. The replication partitions the rate interval at every portfolio and share boundary. On each piece, total profit is a polynomial of degree at most four. Evaluating every real stationary point and every endpoint gives the global best response up to the stated floating-point tolerance.

For finite type mixtures, all type-specific boundaries are included before profits are aggregated. Symmetric candidates include interior stationary roots, portfolio boundaries and strategy endpoints. A candidate is retained only if no piece yields a profitable deviation beyond tolerance. Finite-difference tests separately check the demand Jacobian, the cap derivatives and pass-through. The funding examples use their exact concave value functions and an analogous piecewise calculation. Analytical proofs establish the special-case theorems; numerical tests verify the reported parameter cases and are not offered as computer-assisted proofs for all admissible economies.

10 Empirical implications and the boundary of the analysis

10.1 What must be measured to test the mechanism?

The paper distinguishes changes in deposit pricing from changes in total deposits, customer allocation and payment use. Testing the model therefore requires linked information on balances and bank choice, not a single measure of wallet adoption. The portfolio component concerns how a household changes its deposits at a fixed bank; the switching component concerns which bank receives them. Fee changes and bundled services also matter if the posted interest rate is not the full effective price.

The local Jacobian offers a concrete identification target. Let B_o and B_c denote, respectively, own- and rival-rate effects on one bank's deposits, evaluated at a symmetric environment. Then

$$Q = 2(B_o + B_c), \qquad \frac{M_2}{\tau} = -2B_c, \qquad S = \frac{2B_o}{\mu}. \tag{37}$$

If M_2 is observed, the cross-rate response identifies the reduced-form differentiation scale in this specification. These are model-implied moment restrictions, not an econometric identification result by themselves. Posted rates are endogenous, and own- and rival-rate instruments must have credible exclusions from convenience, perceived safety and common liquidity demand.

One possible design would randomise interest offers among consenting customers while tracking both retained balances and transfers across banks. A complementary design could vary account-switching assistance without changing the wallet balance cap. The first targets rate sensitivity; the second changes relationship frictions. Randomising information about a CBDC is not equivalent to either intervention, because it can affect perceived safety, acceptance and trust at the same time. Survey responses remain informative about stated preferences, not automatically about realised residual demand.

At bank level, pre-existing customer balances could construct exposure to a specified cap scenario. A comparison of high- and low-exposure banks would still require credible counterfactual trends, treatment of anticipation, and controls for simultaneous changes in funding conditions and payment services. Predetermined exposure is not random exposure. Spillovers are especially important: in the model, relative-rate competition moves customers across banks, so a nominally untreated bank can be affected by its rival's response.

A useful empirical test is whether a cap change alters the balance-weighted composition of customers and the response to rival offers. The homogeneous prediction uses $Q = 0$ and $M_2 = \mu^2$. The heterogeneous prediction permits $M_2 > \mu^2$ and can reverse the rate effect. A rise in rates without a rise in S would instead be consistent with the funding-scarcity extension. Separately measuring funding costs and the demand Jacobian is therefore necessary to distinguish the channels.

10.2 What the pilot can and cannot show

The pilot can inform operational and user-experience assumptions, but its selected participants, temporary arrangements and beta instrument do not replicate a general retail-CBDC introduction (ECB 2026c). Even a correctly measured pilot funding flow would need a counterfactual and an external-validity argument before being used to estimate the full instrument's effect.

There is therefore no claim here that pilot participation is an instrument for aggregate CBDC disintermediation. Such identification would require linked funding-chain records and

comparable bank outcomes. The Indian study provides evidence from a live setting, but its institutional context and payment alternatives differ from those envisaged for the euro area (Di Maggio et al. 2026).

10.3 Deposit conversion is not a lending identity

In the household model,

$$\mathbb{E}[d_z] + \mathbb{E}[m_z] = \mathbb{E}[w]. \tag{38}$$

The fixed portfolio imposes aggregate substitution. It does not impose a one-for-one loss of loans. The funding extension shows precisely how loan curvature and wholesale supply divide the adjustment.

There is also a balance-sheet distinction. When a bank settles a conversion of deposits into CBDC using reserves, its deposits and reserves decline together; Eurosystem reserve liabilities fall while CBDC liabilities rise. No expansion of the central bank's total balance sheet is required by that transaction alone. Replacement refinancing or subsequent asset transactions can produce different outcomes. This conditional accounting is consistent with the distinctions in Infante et al. (2022); it does not establish permanent quantitative easing or a necessary contraction of credit.

Joint production of payment, deposit and credit services can create additional effects beyond funding replacement (Kashyap et al. 2002). The separate loan-revenue problem in Section 7 does not model transaction-data synergies, credit lines, merchant acquiring or strategic loan competition. These omissions limit what its lending derivative can explain.

10.4 Stock limits do not settle the question of runs

For a fixed population of eligible users, a cap bounds their total CBDC stock. At a date when balances are $m_i \leq \bar{m}$, unused capacity is $\sum_i(\bar{m} - m_i)$. That quantity bounds additional aggregate holdings at unchanged eligibility and cap. It does not bound cumulative payment flows: spending can free wallet capacity for repeated funding. Nor does it bound every bank's net outflow, because merchant sweeps and transfers can redistribute deposits across banks.

Normal-time demand does not determine run probabilities. A dynamic model would need withdrawal timing, deposit insurance, asset liquidation, refinancing and beliefs about solvency. Ahnert et al. (2023) explicitly models opposing withdrawal and bank-response effects of CBDC; Egan et al. (2017) links deposit demand and endogenous bank distress. The static model here omits those mechanisms. A lower bank profit or a larger response to an offer cannot be relabelled as a measured increase in systemic fragility.

These boundaries preserve the relationship to the original dissertation while narrowing the claims. Competition in deposits, competition in payment services and financial stability are related questions, but a derivative derived in one market is not a proof about all three. A fuller quantitative assessment would need account-level data, estimated preferences, bank funding constraints and a dynamic model of stress.

11 Conclusion

A capped public-money balance and a public alternative to banking have different competitive effects. When households retain their bank accounts, residual deposit demand depends on both

portfolio adjustment and the movement of customers between banks. The second moment of balances enters because larger accounts are more sensitive to rate differences and bring more funding when they move.

This yields conditional conclusions rather than a universal rate prediction. With homogeneous households and binding caps, reduced balances weaken bank switching and lower deposit rates. The result is a global equilibrium and survives random-utility bank choice. Sufficient wealth dispersion can reverse it through customer composition. Endogenous wholesale funding introduces a distinct scarcity effect, which can also raise rates even when portfolio responsiveness is zero. The same model separates those price effects from lending adjustment, pass-through and household surplus.

For the digital euro, the immediate contribution is a set of mechanisms and observable distinctions. The relevant evidence concerns own- and cross-bank rate responses, the distribution of account balances, actual wallet funding and banks' replacement costs. Neither a pilot payment nor a simulated outflow is a substitute for those observations. The analysis provides a reproducible theoretical basis for that empirical work without claiming to forecast an issued digital euro or to choose its holding limit.

A Portfolio regularity and aggregation

For fixed z, write $f_z(m;r)$ for the objective in (1). Its derivatives are

$$f_{z,m} = g_z(m;r), \qquad f_{z,mm} = \theta v''(w-m) + \phi v''(m) = -K_z < 0.$$

The feasible interval is compact and non-empty. Existence follows from continuity and uniqueness from strict concavity. The signs of $f_{z,m}$ at its endpoints give (3). With an interior solution, differentiating $g_z(m_z;r) = 0$ yields

$$-1 - K_z m_{z,r} = 0, \qquad 1 - K_z m_{z,r^m} = 0.$$

Using $d_z = w - m_z$ gives (5). A strict corner persists under a small parameter change, yielding the stated zero or unit derivatives there.

The curvature bound makes the interior rate response no larger than $1/\underline{K}$. Bounded wealth also bounds the derivative of the type-specific share near symmetry. Except at a set of portfolio thresholds of measure zero, the product rule is valid type by type and dominated convergence gives Proposition 1. If a positive mass of households lies at a threshold, one-sided derivatives may differ and the smooth formula must be replaced accordingly. Finite-type numerical examples handle such thresholds as candidate rates, not as differentiable points.

For a continuous type distribution, the portfolio-response moment is

$$Q(r) = \int_{\{z:0<m_z(r)<\bar{m}\}} K_z(r)^{-1}\,\mathrm{d}F(z).$$

Changing a cap can alter both the set of interior types and their curvature. There is no sign theorem for the derivative of this integral from concavity alone. The assumption that S is continuously differentiable in (12) is consequently stronger than uniqueness of individual portfolios and is stated separately.

B Global cap equilibria

B.1 Proof of Proposition 2

Condition (13) makes $g(\bar{m};r) \geq 0$ for every $r \leq a$, hence $m = \bar{m}$ throughout the strategy set. All non-interest terms in $V(r)$ are constant, so $V(r) - V(r^*) = d(r - r^*)$. Against a rival's rate r^*, profit is

$$\pi(r, r^*) = (a - r)d\ \mathrm{clip}_{[0,1]}\left[\frac{1}{2} + \frac{d(r - r^*)}{2\tau}\right].$$

If $ad > \tau$, set $r^* = a - \tau/d$. On the interval of interior customer shares, profit is a strictly concave quadratic with second derivative $-d^2/\tau$. Its derivative vanishes at r^*. Zero-share deviations earn zero. Full capture requires $r \geq r^* + \tau/d = a$, which is possible within the strategy set only at a, also with zero profit. Equilibrium profit is $\tau/2 > 0$, so no deviation is profitable.

If $ad \leq \tau$, set the rival rate to zero. An own rate in $[0, a]$ never reaches full capture except possibly at a. On the interior-share interval, profit has non-positive derivative at zero and strictly negative second derivative. Thus zero is a global best response. Its profit is $ad/2$.

To establish uniqueness among symmetric equilibria, any interior symmetric rate must solve $a - r = \tau/d$. A symmetric rate at a cannot be optimal because a sufficiently small cut gives a positive share and positive profit. At zero, the right derivative is $-d/2 + ad^2/(2\tau)$, which is positive precisely when the interior solution is feasible. These cases exhaust symmetric candidates. No claim about absence of asymmetric equilibria is needed. Differentiating the positive-rate formula proves (15); below its boundary, the zero-rate branch is not continued as a negative rate.

B.2 Proof of Proposition 3

Under global cap binding, $d_z = w_z - \bar{m}$ is independent of the chosen rates. At symmetry, $Q = 0$ and $M_2 = \mu^2 + v_w$. Proposition 1 then yields the candidate (17).

For a deviation $r \in [0, r^*]$, condition (18) implies

$$\frac{1}{2} + \frac{d_z(r - r^*)}{2\tau} \geq 0$$

for every type. No type has full share on this interval. The aggregate profit is therefore a quadratic with second derivative $-M_2/\tau < 0$ and derivative zero at r^*. No downward deviation is better.

For $r \in [r^*, a]$, type-specific shares start at one-half and can only remain interior or become one. In an interior segment, type-specific profit has second derivative $-d_z^2/\tau$. At the point of full capture, its slope changes from

$$-d_z + (a - r)\frac{d_z^2}{2\tau} \quad \text{to} \quad -d_z.$$

The slope weakly decreases because $r \leq a$. Each type's profit is thus concave on this upper interval; integrating preserves concavity. Since aggregate profit has derivative zero at r^*, no upward deviation is better.

The right derivative at a symmetric zero rate is $(-\mu + aM_2/\tau)/2 > 0$, since the candidate is positive. A symmetric rate at a is dominated by a small reduction. Every other symmetric rate must solve the same interior condition, which has only one solution because its moments are rate-invariant. Differentiation of (17), using $\mu_{\bar{m}} = -1$ and constant v_w, gives (19).

B.3 Surplus and balance-dependent costs

At symmetric offers, households choose their nearer bank and

$$\mathbb{E}[\min\{\tau x, \tau(1 - x)\}] = 2\int_0^{1/2} \tau x \, \mathrm{d}x = \tau/4.$$

Differentiating (20), with $d_{\bar{m}} = -1$ and $r^*_{\bar{m}} = -\tau/d^2$, gives

$$\mathcal{U}' = dr^*_{\bar{m}} + g(\bar{m}; r^*) = -\tau/d + r^m - r^* - \theta v'(d) + \phi v'(\bar{m}).$$

Substitution of $r^* = a - \tau/d$ yields (21). This calculation keeps τ fixed and refers to the positive-rate branch.

For the alternative cost scale $\tau_0 d^\eta$, $\eta \geq 0$, a household at distance $y \in [0, 1]$ has the additional portfolio derivative $\eta\tau_0(w - m)^{\eta-1}y \geq 0$. Since $g(m; r) \geq g(\bar{m}; a) \geq 0$ for every $m \leq \bar{m}$ and

$r \le a$, the cap remains optimal even after accounting for that cost. Once constrained, d is constant over rate deviations and the same pricing proof applies with $\tau = \tau_0 d^\eta$. The reported derivative follows on an interior neighbourhood.

C Random-utility bank choice with joint wallet use

Independent type-I extreme-value shocks of scale σ give conditional bank shares

$$s_j(z) = \frac{\exp(V_z(r_j)/\sigma)}{\sum_{k=1}^{N} \exp(V_z(r_k)/\sigma)}.$$

At symmetry $s_j = 1/N$, and $s_{j,r_j} = s_j(1 - s_j)d_z/\sigma$. Consequently,

$$D_j = \frac{\mu}{N}, \qquad D_{j,r_j} = \frac{Q}{N} + \frac{N-1}{N^2\sigma} M_2.$$

Their ratio is (22). A common rate change leaves all bank shares unchanged, consistently with the separate portfolio component.

With homogeneous globally capped portfolios, d is fixed. For any rivals' offers, the derivative of own profit has the sign of

$$T(r) = (a - r)\frac{d}{\sigma}(1 - s_j(r)) - 1.$$

For $r \le a$,

$$T'(r) = -\frac{d}{\sigma}(1 - s_j) - (a - r)\frac{d^2}{\sigma^2} s_j(1 - s_j) < 0.$$

Thus an interior zero is the unique global maximiser on $[0, a]$. At symmetric shares, $T = 0$ yields (23). Its positivity assumption places the root inside the strategy set; the derivative sign verifies all own-rate deviations. The same monotonicity and the symmetric first-order condition give uniqueness among symmetric equilibria. Profit and the cap derivative follow by substitution.

D The exclusive outside-option benchmark

The logit shares are

$$s_j = \frac{e^{(q+\ell r_j)/\sigma}}{e^{A_0/\sigma} + e^{(q+\ell r_A)/\sigma} + e^{(q+\ell r_B)/\sigma}}.$$

Set $\alpha = \ell/\sigma$. At symmetry the own-rate derivative is $\alpha s(1 - s)$, whereas a common bank-rate change gives $s_r = \alpha s s_0$. At a fixed common rate, $s_{A_0} = -s s_0/\sigma$. The symmetric first-order function is

$$H(r, A_0, a) = r + \frac{1}{\alpha(1 - s)} - a.$$

It satisfies

$$H_r = 1 + \frac{s s_0}{(1 - s)^2} > 0, \qquad H_{A_0} = -\frac{s s_0}{\ell(1 - s)^2}, \qquad H_a = -1.$$

Since $s < 1/2$ and $a > 2/\alpha$, $H(0) < 0 < H(a)$. Hence there is a unique symmetric root. For fixed rival and public offers, the own-profit derivative has the sign of $\alpha(a - r)(1 - s_j) - 1$,

whose derivative is strictly negative over $[0, a]$, as in Appendix C. The root is therefore a global best response.

Implicit differentiation gives (25). The total share derivative is

$$\frac{\mathrm{d}s}{\mathrm{d}A_0} = -\frac{ss_0}{\sigma}\frac{(1-s)^2}{(1-s)^2 + ss_0} < 0.$$

Equilibrium profit $\ell s(a-r) = \sigma s/(1-s)$ consequently falls. Up to a constant independent of A_0, expected household utility is

$$\sigma \log\left(2e^{(q+\ell r^*)/\sigma} + e^{A_0/\sigma}\right),$$

whose total derivative is $s_0 + 2s\ell r^*_{A_0} > 0$. These are comparative statics of an exclusive option. A restriction $\bar{m} < \ell$ would remove that allocation from the choice set and is not represented by the preceding quality derivative.

E Funding and lending equilibrium

The first-order condition for (26) is $R'(L) = C'(L-D)$. Its derivative gives $L_D = C''/(C''-R'')$ and $F_{b,D} = R''/(C'' - R'')$. Applying the envelope theorem establishes (27). The denominator in (29) follows by differentiating

$$r + S(r, \bar{m})^{-1} - \lambda(\mu(r, \bar{m})/2) = 0.$$

Both S and μ in this expression vary with a common symmetric rate; the underlying first-order condition still uses a unilateral demand derivative.

For the quadratic specification, substitution into $R' = C'$ gives (31). The condition $A - c_0 > kd$ ensures $F_b(D) > 0$ on the entire attainable interval $[0, d]$, including deviations. Since $\lambda(D) = \lambda_0 - bD$, $\Psi'' = -b < 0$ there.

Under global cap binding, the bank's demand against r^* is

$$D(r) = d\ \mathrm{clip}_{[0,1]}\left[\frac{1}{2} + \frac{d(r-r^*)}{2\tau}\right].$$

Within the interior-share interval, set $q_D = d^2/(2\tau) > 0$. Then

$$\pi'(r) = [\lambda(D(r)) - r]q_D - D(r), \qquad \pi''(r) = -bq_D^2 - 2q_D < 0.$$

At $r^* = \lambda(d/2) - \tau/d$, the derivative is zero. This rate is positive by assumption and below λ_0. A zero-share region yields the constant profit $\Psi(0)$ and meets the interior-share objective continuously; the strict interior maximum dominates it whenever that region is attainable. A full-share region yields $\Psi(d) - rd$, which decreases in r and also meets the interior-share objective continuously. It cannot improve on the interior maximum. Thus the rate is a global best response.

At a symmetric zero rate, the right derivative is positive under $d\lambda(d/2) > \tau$. At a symmetric upper-end rate λ_0, it is negative, so a small reduction raises profit. Every symmetric interior rate satisfies the unique equation (32). Differentiating that equation and the optimal funding quantities, with $D^* = d/2$ and $D^*_{\bar{m}} = -1/2$, proves (33).

F Quadratic portfolio branches and numerical completeness

For quadratic convenience, $g(m;r) = C - r - Bm$, giving (35). On a homogeneous interior branch, put $b_p = 1/B$ and $\alpha_p = w - C/B$. Then

$$d = \alpha_p + b_p r, \qquad S = \frac{b_p}{d} + \frac{d}{\tau}, \qquad S_r = b_p \left(\frac{1}{\tau} - \frac{b_p}{d^2} \right).$$

Write the symmetric stationarity equation in deposits rather than the rate:

$$G(d) = \frac{d - \alpha_p}{b_p} + \frac{\tau d}{b_p \tau + d^2} - a = 0.$$

With $u = d^2/(b_p\tau)$,

$$G'(d) = \frac{1}{b_p} \frac{u^2 + u + 2}{(1+u)^2} > 0.$$

Hence there is at most one stationary symmetric root within this branch, and its local denominator in (12) is positive. Since $S_r/S^2 = (u-1)/(1+u)^2$, substitution into that equation yields the first identity in (36). On the same branch $d_{r^m} = -b_p$, hence $S_{r^m} = -S_r$ and $r^*_{r^m} = 1 - \beta$.

The derivative of $\beta(u)$ has the sign of $-u^2 + 2u + 3 = -(u-3)(u+1)$. Its minimum is approached as $u \downarrow 0$, giving $1/2$; its maximum occurs at $u = 3$, giving $8/7$. This proves Proposition 5. No global optimality is inferred from the monotonicity of G.

For numerical verification, a finite-type distribution is represented by positive weights summing to one. Portfolio changes occur at $r = C_z - B_z \bar{m}$ and $r = C_z$. Splitting the feasible rate interval at all such points makes each $d_z(r)$ affine or constant and each $V_z(r)$ a polynomial of degree at most two. For a fixed rival rate, split further at all real roots of $V_z(r) - V_z(r^*) = \pm\tau$. Each clipped share is then either zero, one or a known quadratic on every piece. Deposit demand has degree at most three and profit has degree at most four.

The replication evaluates every piece endpoint and every real root of the profit derivative inside its piece. It also verifies the polynomial representation against direct household and bank calculations. Symmetric candidates are generated from the piecewise polynomial equation

$$(a - r) \left[\mathbb{E}[d_{z,r}] + \frac{\mathbb{E}[d_z^2]}{\tau} \right] - \mathbb{E}[d_z] = 0,$$

plus portfolio thresholds and the strategy endpoints. Candidates need not be differentiable to undergo the global check. The tolerance used for profitable deviations is reported in the replication output; exact-arithmetic proof is supplied separately for the analytical propositions, not claimed for floating-point root finding.

G Notation and interpretation

Symbol	Meaning
w, θ, ϕ	Liquid wealth and convenience weights; components of household type z.
m, d	CBDC and bank deposits in the earmarked liquid portfolio, with $d + m = w$.
$\bar{m}, r^m$	Individual CBDC holding limit and its net rate; the baseline has $r^m = 0$.
r_A, r_B, r^*	Bank deposit offers and a common equilibrium offer.
a	Constant marginal private value of deposit funding in the baseline.
τ	Scale of spatial bank differentiation, measured in household utility units.
V_z, s_j, D_j	Indirect portfolio value, type-specific bank share and bank deposits.
μ, Q, M_2	Mean deposits, mean portfolio rate response and second deposit moment.
S	Own-rate semi-elasticity of a bank's residual deposit demand at symmetry.
L, F_b, Ψ, λ	Loans, wholesale funding, bank value before deposit interest and its derivative.
A, k, c_0, χ	Quadratic lending and wholesale-cost parameters.
σ, N	Taste-shock scale and number of banks in the logit extension.
ℓ, A_0, α	Fixed bundle, public-option quality and rate sensitivity in exclusive choice.
B, C, b_p, u	Quadratic portfolio coefficients, interior balance slope and curvature ratio.

A rate is a net return per model period. A balance is measured in a common model numeraire; numerical balance units are not euros. An "interior portfolio" means $0 < m < \bar{m}$, while an "interior bank rate" means the offer is strictly inside its rate-choice interval. These are independent conditions. All uniqueness statements refer to symmetric equilibria unless explicitly stated otherwise.

References

Ahnert, T., P. Hoffmann, A. Leonello, and D. Porcellacchia (2023). *Central Bank Digital Currency and Financial Stability*. Working Paper Series 2783. Revised June 2024. European Central Bank. URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2783~0af3ad7576.en.pdf.

Albertazzi, U., F. Faber, A. Gavazza, O.-M. Georgescu, and E. Lecomte (July 2026). *Bank Deposit Pricing in the Euro Area*. Working Paper Series 3255. European Central Bank. URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp3255~7622aab746.en.pdf.

Andolfatto, D. (2021). "Assessing the Impact of Central Bank Digital Currency on Private Banks". In: *The Economic Journal* 131.634, pp. 525–540. DOI: 10.1093/ej/ueaa073.

Auer, S., N. Branzoli, G. Ferrero, A. Ilari, F. Palazzo, and E. Rainone (Feb. 2024). *CBDC and the Banking System*. Questioni di Economia e Finanza 829. Banca d'Italia. URL: https://www.bancaditalia.it/pubblicazioni/qef/2024-0829/index.html?com.dotmarketing.htmlpage.language=1.

Bech, M. L. and R. Garratt (Sept. 2017). "Central Bank Cryptocurrencies". In: *BIS Quarterly Review*, pp. 55–70. URL: https://www.bis.org/publ/qtrpdf/r_qt1709f.pdf.

Becker, H. and L. Grabia (2025). "Digital Euro Holding Limits and Monetary Policy Implementation: A Microdata-Based Perspective". In: *International Finance* 28.3, pp. 158–179. DOI: 10.1111/infi.70004.

Chiu, J., S. M. Davoodalhosseini, J. Jiang, and Y. Zhu (2023). "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment". In: *Journal of Political Economy* 131.5, pp. 1213–1248. DOI: 10.1086/722517.

Council of the European Union (Dec. 19, 2025). *Single Currency: Council Agrees Position on the Digital Euro and on Strengthening the Role of Cash*. URL: https://www.consilium.europa.eu/en/press/press-releases/2025/12/19/single-currency-council-agrees-position-on-the-digital-euro-and-on-strengthening-the-role-of-cash/ (visited on 09/18/2026).

Di Maggio, M., P. Ghosh, S. K. Ghosh, N. Vats, and A. Wu (Aug. 2026). *Central Bank Digital Currency (CBDC), Bank Deposits, and Digital Payments: First Evidence from a Live Retail CBDC*. Working Paper 32457. Revision of the paper first issued in May 2024. National Bureau of Economic Research. DOI: 10.3386/w32457.

Drechsler, I., A. Savov, and P. Schnabl (2017). "The Deposits Channel of Monetary Policy". In: *The Quarterly Journal of Economics* 132.4, pp. 1819–1876. DOI: 10.1093/qje/qjx019.

Egan, M., A. Hortaçsu, and G. Matvos (2017). "Deposit Competition and Financial Fragility: Evidence from the US Banking Sector". In: *American Economic Review* 107.1, pp. 169–216. DOI: 10.1257/aer.20150342.

European Central Bank (Oct. 30, 2025a). *Eurosystem Moving to Next Phase of Digital Euro Project*. URL: https://www.ecb.europa.eu/press/pr/date/2025/html/ecb.pr251030~8c5b5beef0.en.html (visited on 09/18/2026).

— (Oct. 30, 2025b). *Preparation Phase of a Digital Euro: Closing Report*. URL: https://www.ecb.europa.eu/euro/digital_euro/progress/html/ecb.deprp202510.en.html (visited on 09/18/2026).

— (Sept. 15, 2026a). *Call for Online and Mobile Merchants to Participate in Digital Euro Pilot Now Open*. URL: https://www.ecb.europa.eu/press/intro/news/html/ecb.mipnews260915.en.html (visited on 09/18/2026).

— (July 14, 2026b). *ECB Selects 36 Payment Service Providers to Join Digital Euro Pilot*. URL: https://www.ecb.europa.eu/press/pr/date/2026/html/ecb.pr260714~8cd07d9d45.en.html (visited on 09/18/2026).

— (2026c). *FAQs on the Digital Euro Pilot*. Pilot overview, especially Questions 1–4. URL: https://www.ecb.europa.eu/euro/digital_euro/pilot/html/ecb.faq-digital-euro-pilot.en.html (visited on 09/18/2026).

European Commission (June 28, 2023). *Proposal for a Regulation on the Establishment of the Digital Euro*. COM(2023) 369 final. Legislative proposal. URL: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52023PC0369 (visited on 09/18/2026).

European Parliament (July 9, 2026). *Digital Euro: MEPs Ready to Start Negotiations*. Press release 20260708IPR46377. URL: https://www.europarl.europa.eu/news/en/press-room/20260708IPR46377/ (visited on 09/18/2026).

Georgarakos, D., G. Kenny, L. Laeven, and J. Meyer (2025). *Consumer Attitudes towards a Central Bank Digital Currency*. Working Paper Series 3035. European Central Bank. URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb~cde4bd616e.wp3035en.pdf.

Hotelling, H. (1929). "Stability in Competition". In: *The Economic Journal* 39.153, pp. 41–57. DOI: 10.2307/2224214.

Infante, S., K. Kim, A. Orlik, A. F. Silva, and R. J. Tetlow (2022). *The Macroeconomic Implications of CBDC: A Review of the Literature*. Finance and Economics Discussion Series 2022-076. Board of Governors of the Federal Reserve System. DOI: 10.17016/FEDS.2022.076.

Kashyap, A. K., R. Rajan, and J. C. Stein (2002). "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking". In: *The Journal of Finance* 57.1, pp. 33–73. DOI: 10.1111/1540-6261.00415.

Keister, T. and D. Sanches (2023). "Should Central Banks Issue Digital Currency?" In: *The Review of Economic Studies* 90.1, pp. 404–431. DOI: 10.1093/restud/rdac017.

Meaning, J., B. Dyson, J. Barker, and E. Clayton (2021). "Broadening Narrow Money: Monetary Policy with a Central Bank Digital Currency". In: *International Journal of Central Banking* 17.2, pp. 1–42. URL: https://www.ijcb.org/journal/ijcb21q2a1.htm.

Ruzzante, L. (2024). "Analisi dell'introduzione dell'euro digitale: prospettive, sfide ed opportunità nell'era dei big data per il sistema bancario europeo". Degree in Economia e Big Data, academic year 2023/2024. Supervisor: Silvia Nenci. In Italian. Undergraduate dissertation. Università degli Studi Roma Tre.

Schaffer, M. and N. Segev (2022). "The Deposits Channel Revisited". In: *Journal of Applied Econometrics* 37.2, pp. 450–458. DOI: 10.1002/jae.2869.

Train, K. E. (2009). *Discrete Choice Methods with Simulation*. 2nd ed. Cambridge University Press. URL: https://eml.berkeley.edu/books/choice2.html.